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A new type of surface polaritons at the interface of the magnetic gyrotropic media

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Abstract

A new type of surface electromagnetic waves localized near the interface of the halves of the same transparent uniaxial magnetic gyrotropic medium is theoretically predicted. The gyration vectors of the halves are perpendicular to the interface and are oppositely directed to each other. Existence of such waves is due to both anisotropy of the medium and non-coincidence of the gyration vector directions on different sides of the interface. Distribution of intensity and variation of polarization of the surface polaritons as a function of the distance from the interface are studied. It is shown that the surface waves under consideration can be excited in uniformly magnetized ferromagnetics with axis of easy magnetization if their frequency does not exceed the magnetic resonance frequency.

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1. Introduction

At present a new kind of surface electromagnetic waves along with familiar surface polariton excitations (surface exitons, surface plasmons, phonon polaritons [1–3]) is being studied. Existence of these waves is the result of anisotropy of border media [4–11]. Such waves were theoretically predicted for the first time by Marchevsky *et al* [4] and D'yakonov [5] and were called singular [4] or dispersionless [8–10] surface waves. Inasmuch as the energy localization of these waves near the interface is due to interaction of inhomogeneous partial waves in an anisotropic medium (in a uniaxial crystal these are ordinary and extraordinary waves) the term 'anisotropy-driven polariton' or simply 'anisotropic polariton' was proposed [7].

The specific feature of the singular surface electromagnetic waves in the general case is impossibility of their propagation in an arbitrary chosen direction along the interface. For example, if a positive uniaxial crystal borders with an isotropic medium and its optical axis is parallel to the interface then the surface waves can travel only in some allowed directions which form sectors in the cut plane [5]. The lesser the anisotropy of the crystal, the smaller the angular width of these sectors. If the uniaxial crystal is negative then the surface waves cannot be excited in any direction along the cut plane. The sectors of the allowed propagation directions of surface waves are typical also for the boundary between a biaxial crystal and an isotropic medium [7, 10, 11] and between two identical positive uniaxial crystals with different orientation of optical axes [6, 8].

We have derived [14] the dispersion dependences for surface polaritons in arbitrary linear bianisotropic media and have proposed the general approach based on the integral representation of the surface wave impedance tensors for contacting media. This approach was earlier worked out by Stroh, Barnett, Lothe and Chadwick in theory of surface acoustical waves in anisotropic media (see [15] and references cited there). Then they applied it to study the surface waves in cubic, hexagonal, orthorhombic and monoclinic elastic materials. Constitutive equations for the monochromatic electromagnetic field with frequency ω in a bianisotropic medium have the form (see [16–18])

$$\boldsymbol{D}(\omega) = \varepsilon(\omega)\boldsymbol{E}(\omega) + \alpha(\omega)\boldsymbol{H}(\omega), \qquad \boldsymbol{B}(\omega) = \beta(\omega)\boldsymbol{E}(\omega) + \mu(\omega)\boldsymbol{H}(\omega), \tag{1}$$

where ε, μ, α, β are three-dimensional complex tensors of the second rank. If the medium is transparent then tensors ε and μ are Hermitian ($\varepsilon^+ = \varepsilon, \mu^+ = \mu$) and $\beta = \alpha^+$ where a sign $^+$ denotes Hermitian conjugation. In the general case, equations (1) involve 72 real parameters (for a transparent medium they involve 36 parameters). Singular surface waves arise in anisotropic transparent media for which equations (1) have the simplest form with $\alpha = \beta = 0$, $\mu = 1$, and the dielectric permittivity tensor ε is symmetric and real (6 parameters, three of them are independent). It is evident that such waves can be excited also in magnetically ordered media (ferromagnetics and antiferromagnetics) when $\alpha = \beta = 0$ and magnetic permeability μ is a complex tensor. Whereas the tensor nature of the permittivity ε becomes apparent as a rule in an optical frequency band the permeability μ becomes a tensor quantity in the radio-frequency band (sometimes for antiferromagnetics in hyperhigh-frequency band). Furthermore in the presence of the external stationary magnetic field or residual magnetization the tensor μ is non-symmetric and for transparent media can be represented by the sum of the symmetric real part and antisymmetric imaginary part. The latter corresponds to special kind of a high-frequency anisotropy of matter—gyrotropy. For uniaxial crystals such representation has the form [16]

$$\mu = \mu_1 + (\mu_2 - \mu_1)\boldsymbol{c} \otimes \boldsymbol{c} + \mathrm{i}g\boldsymbol{c}^\times, \tag{2}$$

where μ_1, μ_2 are scalar parameters, g is a gyration parameter, $\boldsymbol{c}$ is the unit vector directed along the higher order rotational axis of symmetry, $g\boldsymbol{c}$ is the gyration vector, $\boldsymbol{c} \otimes \boldsymbol{c}$ is a dyad (direct product of vectors), $\boldsymbol{c}^\times$ is the antisymmetric tensor dual to the vector $\boldsymbol{c}$ ($\boldsymbol{c}^\times_{ik} = e_{ijk}c_j$, e_{ijk} is the Levi-Civita tensor). In (2) multiplication of the parameter μ_1 by the unit tensor 1 in three-dimensional space is intended.

Properties of magnetic materials and electromagnetic waves in magnetics are the subject of investigation in numerous works and monographs. These materials find various applications in radio electronics and optoelectronics [19]. Comprehensive experimental researches of magnetics were started by Curie [20] who studied the temperature dependency of magnetization of diamagnetic, paramagnetic and ferromagnetic substances. One of the first systematical formulations of a molecular theory of magnetism was given in Bloch's monograph [21]. As is known, consistent and logically complete theory of magnetism is quantal and statistical (see for example, [22–24]). Optical and microwave properties of magnetic materials are being studied, on the one hand, phenomenologically with the use of the constitutive equations of the form of (1) or their relativistic analogues [17, 25] allowing for temporal and spatial dispersion [28, 30] and, on the other hand, on the basis of modern quantum field theory [22, 29]. Note in this connection that the basic equations of the optics of anisotropic magnetic crystals turn out to be similar to the equations which determine light propagation in a gravitational field in vacuum [26, 27]. This analogy enables us to treat the laws of crystal optics geometrically and to consider that light in magnetic anisotropic media propagates along the world lines of zero length as in the vacuum. Magnetic anisotropy reveals itself as a metric property of space.

Electromagnetic waves in magnetically ordered media are usually called magnetic polaritons. The waves which propagate along the interface of two different magnetic media or along the interface of a magnetic medium and vacuum are called surface magnetic polaritons. As a rule, when such surface magnetic polaritons are considered the semi-infinite medium is supposed to be magnetized parallel to the interface [32]. At that the surface polaritons exhibit nonreciprocal properties of a gyromagnetic medium. For the simplest case of the interface of the semi-infinite gyromagnetic medium and vacuum such nonreciprocity consists in very different dispersion relations, and hence velocities and polarizations for the modes propagating in opposite directions $\boldsymbol{k}$ and $-\boldsymbol{k}$ [33].

In the present paper we predict the existence of a special type of surface magnetic polaritons which arise due to the difference in the directions of the gyration vectors of the contacting uniaxial magnetic materials. The boundaries of the two materials are considered, one of them is characterized by the magnetic permeability tensor μ (2) with positive μ_1 and μ_2 and the other by the complex conjugate tensor μ':

$$\mu' = \mu_1 + (\mu_2 - \mu_1)\boldsymbol{c} \otimes \boldsymbol{c} - \mathrm{i}g\boldsymbol{c}^{\times} \tag{3}$$

which corresponds to the opposite direction of the gyration vector. It is assumed that vector $\boldsymbol{c}$ is perpendicular to the interface. The interface can be built up by cutting an unbounded magnetic gyrotropic crystal by the plane perpendicular to $\boldsymbol{c}$ then rotating one of the crystal halves of 180° about an axis in the cut plane and joining the crystal halves. It is clear that at $g = 0$ tensors μ and μ' coincide, i.e. the crystal is completely restored when its parts are joined. So in this case no surface wave excitations are possible. The purpose of the paper is in establishing the relationship between parameters μ_1, μ_2 and $g \neq 0$ such that excitation of the surface magnetic polaritons turns out to be possible.

The paper is organized as follows. In section 2 expressions are given which describe the field distribution of the surface electromagnetic wave at the interface of linear bianisotropic media. General conditions are indicated when the energy of the wave field is localized on both sides of the interface. For that the dimensionless frequency of the wave $\nu = \omega/(ck)$ should not exceed the minimum of two values of the so-called limiting frequencies of body waves in the contacting media [13, 14]. These limiting frequencies can be found geometrically by studying the sections of the refraction surface. The limiting frequencies for body waves in the case of transparent uniaxial magnetic gyrotropic crystals under consideration are calculated in section 3.

The dispersion equation for surface magnetic polaritons is derived in section 4 in two ways. For convenience of the reader not acquainted with the main results of works [13, 14] the dispersion equation is derived in a standard way by starting from Maxwell's equations and boundary conditions. On the other hand, the same equation is obtained on the basis of integral representation of the surface impedance tensors [14] which are calculated in appendix A. Note that there also exist some other approaches to study surface electromagnetic waves in complex structures. In particular, in paper [34] a dispersion equation is proposed along with examination of the existence of its solutions by finding the poles of the complex reflection and transmission coefficients of the electromagnetic waves.

In section 5 the relations between parameters μ_1, μ_2 and g are obtained when the dispersion equation for the surface magnetic polaritons has solutions. It is shown that depending on the gyration parameter g decay of the amplitudes of the inhomogeneous partial waves in each gyrotropic medium when moving away from the interface can be either purely exponential or spatially oscillating. In this section we investigate variation of the polarization of the surface polaritons and distribution of the energy density and energy-flux density depending on the distance from the interface. A practically important case of small gyrotropy is also considered and the penetration depth of surface waves as a function of g is estimated.

In sections 3–5 we do not specify the frequency dependence of inverse dielectric permittivity $a = \varepsilon^{-1}$ and parameters μ_1, μ_2 and g involved in (2) and (3). It is assumed that these parameters are specified at a fixed frequency ω of surface wave in a microwave frequency band. In the final section 6 in the absence of an external stationary magnetic field, the interface of the uniformly magnetized ferromagnetics with axis of easy magnetization is considered for which well-known resonance dependence of μ_1, μ_2 and g on ω [28, 31, 32] is taken into account. For this case, dispersion curves $k = k(\omega)$ at different values of the anisotropy factor of the magnetization energy are plotted.

In this paper, we use the notation of multiplication operations with three-dimensional scalars, vectors and tensors accepted in [11, 14, 16]. The scalar (internal) product of vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ is marked as $\boldsymbol{uv}$, the vector (external) product as $\boldsymbol{u} \times \boldsymbol{v}$ and the tensor product as $\boldsymbol{u} \otimes \boldsymbol{v}$. The scalar product of a tensor β and a vector $\boldsymbol{u}$ is marked as $\beta\boldsymbol{u}$, at that $(\beta\boldsymbol{u})_i = \beta_{ij}u_j$. For vector $\boldsymbol{v}$ and the antisymmetric tensor $\boldsymbol{u}^\times$ dual to vector $\boldsymbol{u}$ the relations $\boldsymbol{u}^\times\boldsymbol{v} = \boldsymbol{u} \times \boldsymbol{v}$, $\boldsymbol{v}\boldsymbol{u}^\times = \boldsymbol{v} \times \boldsymbol{u}$ hold.

2. The field distribution of the surface electromagnetic wave

Surface electromagnetic waves at the interface of arbitrary linear bianisotropic media are described by equations (see, for example, [14])

$$\begin{aligned} \boldsymbol{H}(\boldsymbol{r}, t) &= \sum_{s=1}^{2} C_s \boldsymbol{H}_s^0 \exp[\mathrm{i}k(\boldsymbol{b} + \eta_s \boldsymbol{q})\boldsymbol{r} - \mathrm{i}\omega t], \\ \boldsymbol{E}(\boldsymbol{r}, t) &= \sum_{s=1}^{2} C_s \boldsymbol{E}_s^0 \exp[\mathrm{i}k(\boldsymbol{b} + \eta_s \boldsymbol{q})\boldsymbol{r} - \mathrm{i}\omega t]. \end{aligned} \tag{4}$$

It is supposed that the z-axis of the Cartesian system of coordinates is perpendicular to the interface $z = 0$. In equations (4) k is a wave number and ω is a frequency of the surface wave; the unit normal vector to the interface $\boldsymbol{q}$ is oriented in positive direction; the unit vector $\boldsymbol{b}$ determines the propagation direction of the surface wave ($\boldsymbol{bq} = 0$); C_s are weight factors which characterize the contribution of partial waves and the amplitudes of magnetic and electric fields relating to the interface are $\boldsymbol{H}_s^0$ and $\boldsymbol{E}_s^0$ ($s = 1, 2$). We consider that equations (4) relate to the medium situated in half-space $z < 0$ and in this connection the imaginary parts of the complex decay coefficients η_s are negative. This ensures reduction of the field intensity as we move away from the interface. For the medium in half-space $z > 0$ the field intensities $\boldsymbol{H}'(\boldsymbol{r}, t)$ and $\boldsymbol{E}'(\boldsymbol{r}, t)$ are described by expressions analogous to (4). We mark the appropriate amplitudes, weight factors and decay coefficients as $\boldsymbol{H}_s'^0$, $\boldsymbol{E}_s'^0$, C_s', η_s', and here $\operatorname{Im} \eta_s' > 0$.

The dimensionless reduced frequency $\nu = \omega/(ck)$ represents the phase velocity of the surface wave expressed in units of velocity c of light in vacuum.

In paper [14] a general formalism has been developed which allows uniform derivation of the surface-wave dispersion relations for the interface of bianisotropic media when vectors $\boldsymbol{b}$ and $\boldsymbol{q}$ are arbitrarily oriented with respect to crystallographic symmetry elements of the contacting media. It has been established that the necessary and sufficient condition of nonvanishing imaginary parts of coefficients η_s and η'_s, $s = 1, 2$, is

$$0 \leqslant \nu < \hat{\nu}_{\mathrm{L}} = \min(\nu_{\mathrm{L}}, \nu'_{\mathrm{L}}), \tag{5}$$

where ν_{L} and ν'_{L} are the so-called *limiting frequencies* of body waves in the contacting media. They can be found geometrically with the use of the sections of the refraction surface by the plane passing through vectors $\boldsymbol{b}$ and $\boldsymbol{q}$ (reference plane). Namely, the limiting frequency is the reciprocal of the distance from the reference point O to the straight line that is parallel to vector $\boldsymbol{q}$ and is tangent to the outer section curve of the refraction surface. According to inequality (5) the phase velocity of the surface wave is subluminal, i.e. less than the phase velocities of any body waves travelling along $\boldsymbol{b}$ in each contacting medium.

3. The interface of magnetic gyrotropic media. Refraction surface sections and limiting frequencies

Consider the boundary of two transparent uniaxial magnetic gyrotropic crystals of trigonal, tetragonal or hexagonal symmetry. Let these crystals at frequency ω be characterized by inverse dielectric permittivity $a = \varepsilon^{-1} > 0$ and magnetic permeability tensors μ (2) and μ' (3) with positive μ_1 and μ_2. The normal $\boldsymbol{q}$ to the interface is regarded coincident with $\boldsymbol{c}$. It is clear that in this case all propagation directions $\boldsymbol{b}$ of surface waves along the interface $z = 0$ are equivalent.

To calculate limiting frequencies ν_{L} and ν'_{L} we study refraction surfaces that are the locus of extreme points of the body wave refraction vectors $\boldsymbol{m} = n\boldsymbol{n}$ with origins aligned at a reference point O. Here $\boldsymbol{n}$ is an unit vector of the wave normal and n is a refractive index. Maxwell's equations for body waves in the medium with the magnetic permeability tensor being equal to μ (2) take the form [16]

$$\boldsymbol{m}^{\times}\boldsymbol{E} = \mu\boldsymbol{H}, \qquad a\boldsymbol{m}^{\times}\boldsymbol{H} = -\boldsymbol{E}. \tag{6}$$

Eliminating vector $\boldsymbol{E}$ from (6), we get

$$(a\boldsymbol{m}^{\times}\boldsymbol{m}^{\times} + \mu)\boldsymbol{H} = 0. \tag{7}$$

The equation of the refraction surface (Fresnel equation) is written as $\det(a\boldsymbol{m}^{\times}\boldsymbol{m}^{\times} + \mu) = 0$. Now we choose an orthonormal basis $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$ with $\boldsymbol{e}_3 = \boldsymbol{c}$. In this basis the tensors $\boldsymbol{m}^{\times}$ and μ are represented by matrices

$$\begin{pmatrix} 0 & -m_3 & m_2 \\ m_3 & 0 & -m_1 \\ -m_2 & m_1 & 0 \end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix} \mu_1 & -\mathrm{i}g & 0 \\ \mathrm{i}g & \mu_1 & 0 \\ 0 & 0 & \mu_2 \end{pmatrix}$$

so the equation of the refraction surface takes the form

$$a^2\big(m_1^2 + m_2^2 + m_3^2\big)\big[\mu_1\big(m_1^2 + m_2^2\big) + \mu_2 m_3^2\big] \\ - a\big\{[\mu_1(\mu_1 + \mu_2) - g^2]\big(m_1^2 + m_2^2\big) + 2\mu_1\mu_2 m_3^2\big\} + \mu_2\big(\mu_1^2 - g^2\big) = 0. \tag{8}$$

The gyration parameter g appears in relation (8) quadratically. Consequently, the equation of the refraction surface for body waves in the crystal situated in the half-space $z > 0$ and characterized by the magnetic permeability tensor $\mu' = \mu^*$ is the same. Therefore limiting frequencies ν_{L} and ν'_{L} coincide and to find the quantity $\hat{\nu}_{\mathrm{L}} = \nu_{\mathrm{L}}$ it is sufficient to proceed from equation (8).

As is obvious from (8) the refraction surface is axially symmetric. Consider its section by the reference plane passing through vectors $\boldsymbol{b}$ and $\boldsymbol{q} = \boldsymbol{c}$. Let the components of a vector $\boldsymbol{m}$ lying in this plane be m_b and m_q. Then $m_b^2 = m_1^2 + m_2^2$, $m_q = m_3$ and the equation of the

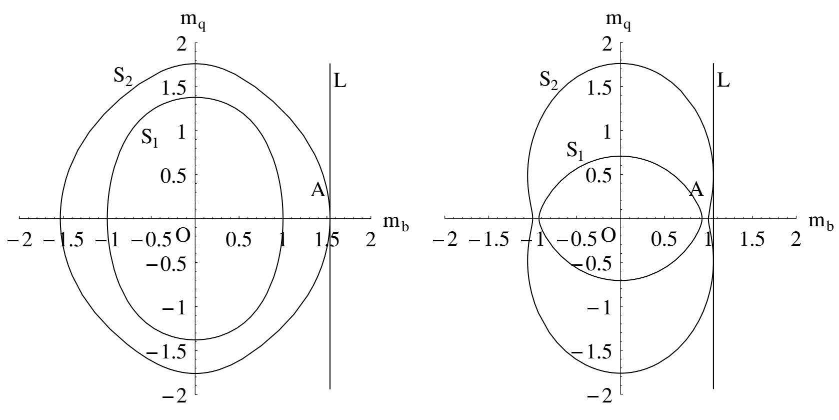

Figure 1. The refraction surface sections by the reference plane: on the left for $a = 1$, $\mu_1 = 2.5$, $\mu_2 = 1$, $g = 0.6$; on the right for $a = 1$, $\mu_1 = 1.8$, $\mu_2 = 1$, $g = 1.3$.

section curves is

$$a^2\left(m_b^2 + m_q^2\right)\left(\mu_1 m_b^2 + \mu_2 m_q^2\right) - a\left\{[\mu_1(\mu_1 + \mu_2) - g^2]m_b^2 + 2\mu_1\mu_2 m_q^2\right\} + \mu_2\left(\mu_1^2 - g^2\right) = 0. \tag{9}$$

Further we transform equation (9), following the technique stated in paper [11] which dealt in particular with the calculation of the limiting frequencies of body waves in biaxial dielectric crystals. We introduce polar coordinates $|\boldsymbol{m}|$ and θ, and find the squared refraction vector

$$\boldsymbol{m}^2 = \frac{1}{2a}(\mu_1\cos^2\theta + \mu_2\sin^2\theta)^{-1}\left\{[\mu_1(\mu_1+\mu_2) - g^2]\cos^2\theta + 2\mu_1\mu_2\sin^2\theta \pm \sqrt{[\mu_1(\mu_1-\mu_2) - g^2]^2\cos^4\theta + 4\mu_2^2 g^2\sin^2\theta}\right\}. \tag{10}$$

Minus and plus signs of the radical in formula (10) correspond to the inner S_1 and outer S_2 section curves, respectively. Returning to variables m_b and m_q, and introducing the notation $\kappa = m_q^2 / m_b^2$, we have for S_2

$$2am_b^2(1+\kappa)(\mu_1 + \mu_2\kappa) = \mu_1(\mu_1+\mu_2) - g^2 + 2\mu_1\mu_2\kappa + \sqrt{[\mu_1(\mu_1-\mu_2) - g^2]^2 + 4\mu_2^2 g^2\kappa(1+\kappa)}. \tag{11}$$

In figure 1 the refraction surface sections are shown that are plotted with the use of formulae (10) for different values of parameters a, μ_1, μ_2, g. It is seen that the part of the outer section curve near the point of intersection A of the curve and the m_b-axis can be either convex or concave. The distance of the straight line L which is tangent to the outer curve and parallel to vector $\boldsymbol{q}$ from reference point O, is the reciprocal of the limiting frequency $1/\nu_{\mathrm{L}}$ and is calculated differently for convexity and concavity. For the part of the curve under consideration the value of κ is small and so we retain in equation (11) the terms which are linear with respect to κ:

$$2am_b^2[\mu_1 + (\mu_1+\mu_2)\kappa)] = \mu_1(\mu_1+\mu_2) - g^2 + 2\mu_1\mu_2\kappa + |\mu_1(\mu_1-\mu_2) - g^2| + \frac{2\mu_2^2 g^2\kappa}{|\mu_1(\mu_1-\mu_2) - g^2|}. \tag{12}$$

Equation (12) is stated in different ways depending upon how quantities g^2 and $\mu_1(\mu_1-\mu_2)$ are ordered:

$$\frac{a\mu_1}{\mu_1^2-g^2}m_b^2+\frac{a\left[\mu_1^2(\mu_1-\mu_2)-g^2(\mu_1+\mu_2)\right]}{(\mu_1^2-g^2)[\mu_1(\mu_1-\mu_2)-g^2]}m_q^2=1, \qquad \text{if} \quad g^2<\mu_1(\mu_1-\mu_2), \tag{13}$$

$$\frac{a}{\mu_2}m_b^2+\frac{a(\mu_2-\mu_1)}{g^2-\mu_1(\mu_1-\mu_2)}m_q^2=1, \qquad \text{if} \quad g^2>\mu_1(\mu_1-\mu_2). \tag{14}$$

Thus, in the linear approximation with respect to κ the part of the outer section curve near m_b-axis is a conic section—ellipse or hyperbola ($g^2\neq\mu_1(\mu_1-\mu_2)$).

For the case $g^2=\mu_1(\mu_1-\mu_2)$ equations (12), (13) and (14) are inapplicable and a separate consideration is needed. Equation (9) takes the form

$$a\left(m_b^2+m_q^2\right)\left(a\mu_1m_b^2+a\mu_2m_q^2-2\mu_1\mu_2\right)+\mu_1\mu_2^2=0.$$

Using an expansion $m_b=b_0+b_1m_q+b_2m_q^2+\cdots$ in a power series with respect to small m_q, we obtain

$$m_q\approx\pm2\sqrt{\frac{\mu_1}{\mu_1-\mu_2}}\left(m_b-\sqrt{\frac{\mu_2}{a}}\right).$$

So the parts of the section curves S_1 and S_2 near m_b-axis are straight lines, and point A is a point of intersection of S_1 and S_2.

Let $\mu_1<\mu_2$. In this case the equation of the part of the outer curve is (14) for arbitrary values of g. It describes an ellipse, since the co-factor of m_q^2 is positive (i.e. the part of the curve S_2 is convex, and line L is tangent to the curve at point A). The length of the line segment OA is a semi-axis of the ellipse equal to $\sqrt{\mu_2/a}$. Consequently, the limiting frequency is determined by $\nu_{\mathrm{L}}=\sqrt{a/\mu_2}$.

Now turn to the contrary case $\mu_1>\mu_2$. Assume the relation $g^2<\mu_1^2(\mu_1-\mu_2)/(\mu_1+\mu_2)<\mu_1(\mu_1-\mu_2)$. Then in equation (13) the co-factor of m_q^2 is positive, and this equation describes the ellipse with semi-axis OA of length $\sqrt{\left(\mu_1^2-g^2\right)/(a\mu_1)}$. Consequently, the limiting frequency is equal to $\nu_{\mathrm{L}}=\sqrt{a\mu_1/\left(\mu_1^2-g^2\right)}$.

If $g^2>\mu_1^2(\mu_1-\mu_2)/(\mu_1+\mu_2)$, then depending on whether the parameter g^2 is less than $\mu_1(\mu_1-\mu_2)$ or not, one should consider either equation (13) or equation (14). In both cases the part of the outer section curve close to the point A is a hyperbola (i.e. it is concave). This implies that the straight line L does not pass A and is tangent to the outer curve at two points. Here the calculation of the quantity ν_{L} is more complicated. For contact points the relation $\mathrm{d}m_b/\mathrm{d}m_q=0$ holds. Supposing that $m_q\neq0$ for these points, we differentiate both sides of equation (9) with respect to m_q, considering coordinate m_b as an implicit function of m_q. Then we set derivatives $\mathrm{d}m_b/\mathrm{d}m_q$ to zero. As a result we obtain the following relation between the coordinates m_q and m_b of the contact points:

$$m_q^2=\frac{\mu_1}{a}-\frac{\mu_1+\mu_2}{2\mu_2}m_b^2. \tag{15}$$

Substituting (15) into (9), we have

$$\tfrac{1}{4}a^2(\mu_1-\mu_2)^2m_b^4-a\mu_2g^2m_b^2+\mu_2^2g^2=0.$$

Since $\nu_{\mathrm{L}}=1/m_b$, the limiting frequency ν_{L} satisfies the equation

$$\mu_2^2g^2\nu_{\mathrm{L}}^4-a\mu_2g^2\nu_{\mathrm{L}}^2+\tfrac{1}{4}a^2(\mu_1-\mu_2)^2=0. \tag{16}$$

Equation (16) has a solution

$$\nu_{\mathrm{L}}^2\equiv\widetilde{\nu}_{\mathrm{L}}^2=\frac{a}{2\mu_2}\left[1+\frac{1}{|g|}\sqrt{g^2-(\mu_1-\mu_2)^2}\right]. \tag{17}$$

Table 1. Limiting frequencies ν_L for surface electromagnetic waves in magnetic gyrotropic crystals.

Case	Relations between μ_1 and μ_2	Parameter g	Limiting frequency ν_L
(i)	$\mu_1 < \mu_2$	Any	$\nu_L = \sqrt{\frac{a}{\mu_2}}$
(ii(a))	$\mu_1 > \mu_2$	$g^2 \leqslant \frac{\mu_1^2(\mu_1-\mu_2)}{(\mu_1+\mu_2)}$	$\nu_L = \sqrt{\frac{a\mu_1}{(\mu_1^2-g^2)}}$
(ii(b))	$\mu_1 > \mu_2$	$g^2 > \frac{\mu_1^2(\mu_1-\mu_2)}{(\mu_1+\mu_2)}$	$\nu_L = \widetilde{\nu}_L$ (formula (17))

In (17) we choose the plus sign of the radical to ensure smooth joining of solution (17) and the solutions for ν_L that have been found earlier. Indeed, choosing such a sign, from (17) at $g^2 = \mu_1^2(\mu_1 - \mu_2)/(\mu_1 + \mu_2)$ we arrive at $\widetilde{\nu}_L^2 = a(\mu_1 + \mu_2)/(2\mu_1\mu_2)$. It is easy to see that the same expression for ν_L^2 at g^2 specified above can be obtained from the earlier derived formula $\nu_L^2 = a\mu_1\big/\left(\mu_1^2 - g^2\right)$.

The values of the limiting frequencies found in this section are listed in table 1.

4. Derivation of the dispersion equation for $\nu = \omega/(ck)$

We shall obtain a dispersion equation for surface electromagnetic waves at the boundary of two media with the magnetic permeability tensors μ (2) and μ' (3) using relations (6) and (7) and the boundary conditions. Each of the inhomogeneous partial waves determining the field distribution (4) is associated with its complex refraction vector

$$\boldsymbol{m}_s = \frac{ck}{\omega}(\boldsymbol{b} + \eta_s\boldsymbol{q}) = \frac{1}{\nu}(\boldsymbol{b} + \eta_s\boldsymbol{q}), \qquad s = 1, 2. \tag{18}$$

Since $\boldsymbol{m}^\times\boldsymbol{m}^\times = \boldsymbol{m}\otimes\boldsymbol{m} - \boldsymbol{m}^2$ [16], from (7) the following equations for the amplitudes $\boldsymbol{H}_s^0$ of the partial waves can be obtained:

$$\left[a\boldsymbol{m}_s\otimes\boldsymbol{m}_s - a\boldsymbol{m}_s^2 + \mu_1 + (\mu_2 - \mu_1)\boldsymbol{c}\otimes\boldsymbol{c} + \mathrm{i}g\boldsymbol{c}^\times\right]\boldsymbol{H}_s^0 = 0.$$

To simplify the following notation we omit the indices s. Let H_b, H_q, H_a be components of the vector $\boldsymbol{H}^0$ in the orthonormal basis consisting of vectors $\boldsymbol{b}$, $\boldsymbol{q}$ and $\boldsymbol{a} = \boldsymbol{b}\times\boldsymbol{q}$. Taking into account that $\boldsymbol{c} = \boldsymbol{q}$, we have the following vector equation:

$$\begin{aligned}&[a\boldsymbol{b}\otimes\boldsymbol{b} + a\eta(\boldsymbol{b}\otimes\boldsymbol{q} + \boldsymbol{q}\otimes\boldsymbol{b}) + a\eta^2\boldsymbol{q}\otimes\boldsymbol{q} - a(1+\eta^2) + \mu_1\nu^2\\&\quad + \nu^2(\mu_2 - \mu_1)\boldsymbol{q}\otimes\boldsymbol{q} + \mathrm{i}g\nu^2\boldsymbol{q}^\times](H_b\boldsymbol{b} + H_q\boldsymbol{q} + H_a\boldsymbol{a}) = 0.\end{aligned} \tag{19}$$

Opening the brackets on the left-hand side of equation (19), we represent it as decomposition with respect to the basis vectors $\boldsymbol{b}$, $\boldsymbol{q}$, $\boldsymbol{a}$ and then set the coefficients of these vectors to zero:

$$\begin{aligned}&-(a\eta^2 - \mu_1\nu^2)H_b + a\eta H_q + \mathrm{i}g\nu^2 H_a = 0, \qquad a\eta H_b - (a - \mu_2\nu^2)H_q = 0,\\&\mathrm{i}g\nu^2 H_b + (a\eta^2 + a - \mu_1\nu^2)H_a = 0.\end{aligned} \tag{20}$$

The system of linear homogeneous equations (20) is compatible if the parameter η satisfies the equation

$$A\eta^4 + B\eta^2 + C = 0, \tag{21}$$

where

$$\begin{aligned}&A = a^2\mu_2, \qquad B = a[a(\mu_1 + \mu_2) - 2\mu_1\mu_2\nu^2],\\&C = \left[a\mu_1 - \left(\mu_1^2 - g^2\right)\nu^2\right](a - \mu_2\nu^2).\end{aligned} \tag{22}$$

In this case one of the components of the vector $\boldsymbol{H}^0$, e.g. H_b, can be taken arbitrarily and the other two are expressed in terms of it:

$$H_q = \frac{a\eta}{a - \mu_2\nu^2} H_b, \qquad H_a = -\frac{\mathrm{i}g\nu^2}{a\eta^2 + a - \mu_1\nu^2} H_b. \tag{23}$$

Note that equation (21) can be straightforwardly obtained from (9) if one puts $m_b = 1/\nu, m_q = \eta/\nu$ according to relations (18). Its solutions $\eta_j = \eta_j(\nu)$, $j = 1, \ldots, 4$, are functions of a dimensionless frequency ν (or velocity in units of c) of the surface wave. According to (5) they are complex if and only if $0 \leqslant \nu < \nu_{\mathrm{L}}$ where limiting frequencies ν_{L} are determined by table 1. At $\nu = \nu_{\mathrm{L}}$ at least two solutions η_j become real and the partial waves corresponding to them will no longer be localized near the interface (see (4)). Equation (4) involves the two roots η_1, η_2 out of the four possible roots of (21), for which the imaginary parts are negative, and so

$$\eta_{1,2}^2 = \frac{1}{2a\mu_2}\left[-a(\mu_1+\mu_2) + 2\mu_1\mu_2\nu^2 \pm \sqrt{a^2(\mu_1-\mu_2)^2 - 4\mu_2 g^2\nu^2(a-\mu_2\nu^2)}\right]. \tag{24}$$

Since equation (21) includes even powers of η, for the two remaining solutions we have $\eta_3 = -\eta_1, \eta_4 = -\eta_2$.

Taking $H_b = (a\eta^2 + a - \mu_1\nu^2)(a - \mu_2\nu^2)$, in accordance with (23) we find the amplitudes of the magnetic field intensity of the partial waves:

$$\boldsymbol{H}_s^0 = \left(a\eta_s^2 + a - \mu_1\nu^2\right)(a - \mu_2\nu^2)\boldsymbol{b} + a\eta_s\left(a\eta_s^2 + a - \mu_1\nu^2\right)\boldsymbol{q} - \mathrm{i}g\nu^2(a - \mu_2\nu^2)\boldsymbol{a}. \tag{25}$$

To calculate the amplitudes of the electric field intensity we use the second equation (6) and substitute (18) and (25) into it:

$$\boldsymbol{E}_s^0 = \mathrm{i}ga\nu\eta_s(a - \mu_2\nu^2)\boldsymbol{b} - \mathrm{i}ga\nu(a - \mu_2\nu^2)\boldsymbol{q} - a\mu_2\nu\eta_s\left(a\eta_s^2 + a - \mu_1\nu^2\right)\boldsymbol{a}. \tag{26}$$

The relations obtained above concern the gyrotropic medium situated in the half-space $z < 0$. When the second contacting medium is under consideration one should replace g by $-g$ throughout. Equation (21) does not change in this regard. But now the solutions of this equation have to be selected which have positive imaginary parts: $\eta_1' = -\eta_1, \eta_2' = -\eta_2$. Therefore the amplitudes of the partial waves are

$$\boldsymbol{H}_s'^0 = \left(a\eta_s^2 + a - \mu_1\nu^2\right)(a - \mu_2\nu^2)\boldsymbol{b} - a\eta_s\left(a\eta_s^2 + a - \mu_1\nu^2\right)\boldsymbol{q} + \mathrm{i}g\nu^2(a - \mu_2\nu^2)\boldsymbol{a}, \tag{27}$$

$$\boldsymbol{E}_s'^0 = \mathrm{i}ga\nu\eta_s(a - \mu_2\nu^2)\boldsymbol{b} + \mathrm{i}ga\nu(a - \mu_2\nu^2)\boldsymbol{q} + a\mu_2\nu\eta_s\left(a\eta_s^2 + a - \mu_1\nu^2\right)\boldsymbol{a}. \tag{28}$$

Further we take into consideration the boundary conditions comprising the continuity of the tangential components of magnetic and electric field at the interface:

$$C_1\boldsymbol{H}_{1\tau}^0 + C_2\boldsymbol{H}_{2\tau}^0 = C_1'\boldsymbol{H}_{1\tau}'^0 + C_2'\boldsymbol{H}_{2\tau}'^0, \qquad C_1\boldsymbol{E}_{1\tau}^0 + C_2\boldsymbol{E}_{2\tau}^0 = C_1'\boldsymbol{E}_{1\tau}'^0 + C_2'\boldsymbol{E}_{2\tau}'^0. \tag{29}$$

In (29) C_1, C_2, C_1', C_2' are weight factors. The tangential amplitude components of the partial waves can be found from formulae (25)–(28) when in these formulae the terms depending on vector $\boldsymbol{q}$ are discarded. Equations (29) may be simplified to

$$\left(a\eta_1^2 + a - \mu_1\nu^2\right)D_1^{(-)} + \left(a\eta_2^2 + a - \mu_1\nu^2\right)D_2^{(-)} = 0, \qquad \eta_1 D_1^{(-)} + \eta_2 D_2^{(-)} = 0, \tag{30}$$

$$D_1^{(+)} + D_2^{(+)} = 0, \qquad \eta_1\left(a\eta_1^2 + a - \mu_1\nu^2\right)D_1^{(+)} + \eta_2\left(a\eta_2^2 + a - \mu_1\nu^2\right)D_2^{(+)} = 0, \tag{31}$$

where we have

$$D_s^{(\pm)} = C_s \pm C_s', \qquad s = 1, 2.$$

With the assumption that $\eta_1 \neq \eta_2$, system (30) has non-trivial solutions $D_1^{(-)}$ and $D_2^{(-)}$ if

$$a - \mu_1 \nu^2 - a\eta_1\eta_2 = 0. \tag{32}$$

Analogously, system (31) has non-trivial solutions $D_1^{(+)}$ and $D_2^{(+)}$ if

$$a - \mu_1 \nu^2 + a\left(\eta_1^2 + \eta_2^2 + \eta_1\eta_2\right) = 0. \tag{33}$$

It is evident that equations (32) and (33) are incompatible. Only one of these equations can be fulfilled, so the following two cases are possible.

(I) Equation (32) holds. Then $D_1^{(+)} = D_2^{(+)} = 0$, i.e. $C_1' = -C_1$, $C_2' = -C_2$, and at the same time $C_2 = -C_1\eta_1/\eta_2$.

(II) Equation (33) holds. Then $D_1^{(-)} = D_2^{(-)} = 0$, i.e. $C_1 = C_1' = -C_2 = -C_2'$.

In both cases the absolute values of the coefficients C_1 and C_1' (C_2 and C_2') are equal. The magnetic permeability tensors μ and μ' of the contacting media differ in the sign of $\mathrm{i}g\boldsymbol{q}^{\times}$ only. By virtue of this symmetry, the field distribution on both sides of the interface is determined by the same absolute-value weights of partial waves.

The complex decay coefficients η_1 and η_2 can either be pure imaginary or can have nonzero real parts (for the latter case $\eta_2 = -\eta_1^*$). This question will be discussed in detail in the next section. Here it is essential that $\eta_1\eta_2 < 0$. Applying the quadratic formula to the equation (21), we have

$$\begin{aligned} \eta_1\eta_2 &= -\sqrt{\frac{C}{A}} = -\frac{1}{a\sqrt{\mu_2}}\sqrt{\left[a\mu_1 - \left(\mu_1^2 - g^2\right)\nu^2\right](a - \mu_2\nu^2)}, \\ \eta_1^2 + \eta_2^2 &= -\frac{B}{A} = -\frac{1}{a\mu_2}[a(\mu_1 + \mu_2) - 2\mu_1\mu_2\nu^2]. \end{aligned} \tag{34}$$

As a result we arrive at the following dispersion equation for surface polaritons at the interface of magnetic gyrotropic media:

$$f(\nu)h(\nu) = 0, \tag{35}$$

where

$$\begin{aligned} f(\nu) &= \sqrt{\mu_2}(a - \mu_1\nu^2) + \sqrt{\left[a\mu_1 - \left(\mu_1^2 - g^2\right)\nu^2\right](a - \mu_2\nu^2)}, \\ h(\nu) &= \mu_1\sqrt{a - \mu_2\nu^2} + \sqrt{\mu_2\left[a\mu_1 - \left(\mu_1^2 - g^2\right)\nu^2\right]} \end{aligned} \tag{36}$$

are functions which, up to a constant, follow from the left-hand side of equations (32) and (33), when relations (34) are substituted.

Function $h(\nu)$ is strictly positive at any value of ν in the interval $[0, \nu_{\mathrm{L}})$, whichever are the values of limiting frequencies ν_{L} enumerated in table 1. Therefore in fact the dispersion equation is set down in the form $f(\nu) = 0$. Thus, only one case (I) of two, (I) and (II), can be realized.

Let us mention another method of derivation of equation (35) with the use of the surface impedance tensors γ and γ' for waves in contacting media. These tensors constrain the resulting electric and magnetic fields at the interface. With the use of boundary conditions, the dispersion equation can be obtained [35, 36] in the form

$$(\overline{\gamma - \gamma'})_t = 0, \tag{37}$$

where subscript t marks a trace of tensor and $\overline{\gamma - \gamma'}$ is the adjoined tensor of $\gamma - \gamma'$ [16].

In appendix A with the use of the general integral formalism in the theory of surface polaritons, worked out in [14], tensors γ and γ' are obtained for surface waves in the magnetic gyrotropic crystals under consideration:

$$\gamma = \frac{\mathrm{i}\sqrt{a}}{\nu\sqrt{\Delta(\nu)}}\left[h(\nu)\nu^2\sqrt{\frac{\mu_2}{a - \mu_2\nu^2}}\boldsymbol{b}\otimes\boldsymbol{b} + \mathrm{i}g\nu^2\sqrt{\mu_2}(\boldsymbol{b}\otimes\boldsymbol{a} - \boldsymbol{a}\otimes\boldsymbol{b}) - f(\nu)\boldsymbol{a}\otimes\boldsymbol{a}\right], \tag{38}$$

$$\gamma' = -\frac{\mathrm{i}\sqrt{a}}{\nu\sqrt{\Delta(\nu)}}\left[h(\nu)\nu^2\sqrt{\frac{\mu_2}{a-\mu_2\nu^2}}\boldsymbol{b}\otimes\boldsymbol{b} - \mathrm{i}g\nu^2\sqrt{\mu_2}(\boldsymbol{b}\otimes\boldsymbol{a}-\boldsymbol{a}\otimes\boldsymbol{b}) - f(\nu)\boldsymbol{a}\otimes\boldsymbol{a}\right], \tag{39}$$

where

$$\Delta(\nu) = a(\mu_1+\mu_2) - 2\mu_1\mu_2\nu^2 + 2\sqrt{\mu_2\left[a\mu_1 - \left(\mu_1^2-g^2\right)\nu^2\right](a-\mu_2\nu^2)}. \tag{40}$$

Substituting equations (38) and (39) into (37), we obtain

$$\frac{4a}{\Delta(\nu)}\sqrt{\frac{\mu_2}{a-\mu_2\nu^2}}f(\nu)h(\nu) = 0. \tag{41}$$

It is easy to show that the function $\Delta(\nu)$ is equal to $(B+2\sqrt{AC})/a$ (see (22)) and is strictly positive at any values of ν in the interval $[0, \nu_{\mathrm{L}})$ for all cases enumerated in table 1. So equation (41) in fact reduces to (35).

5. Analysis of solutions of the dispersion equation. Existence conditions of surface polaritons

As was shown in section 4 the dispersion equation for surface electromagnetic waves at the interface of magnetic gyrotropic media with magnetic permeability tensors μ (2) and μ' (3), $\boldsymbol{c} = \boldsymbol{q}$ is written as $f(\nu) = 0$ or, in expanded form

$$\sqrt{\mu_2}(a-\mu_1\nu^2) + \sqrt{\left[a\mu_1 - \left(\mu_1^2-g^2\right)\nu^2\right](a-\mu_2\nu^2)} = 0. \tag{42}$$

Rationalizing (42), we obtain the equation

$$\mu_2 g^2\nu^4 + a[\mu_1(\mu_1-\mu_2) - g^2]\nu^2 - a^2(\mu_1-\mu_2) = 0 \tag{43}$$

and formally find its presumably nonnegative solution $\nu^2 = \nu_{\mathrm{S}}^2$

$$\nu_{\mathrm{S}}^2 = \frac{a}{2\mu_2 g^2}\left\{\sqrt{[\mu_1(\mu_1-\mu_2)+g^2]^2 - 4g^2(\mu_1-\mu_2)^2} - \mu_1(\mu_1-\mu_2) + g^2\right\} \tag{44}$$

which corresponds to the squared reduced frequency of the surface wave. It is of major importance to search for those values of the material parameters a, μ_1, μ_2 and g when equation (42) has solution $\nu = \nu_{\mathrm{S}}$ in the interval $[0, \nu_{\mathrm{L}})$ (i.e. when parameters η_s (24) are complex, and the energy of the wave is localized near the interface). Note that $f(\nu)$ in (36) is a decreasing function and it is positive at the left-hand end point of the interval: $f(0) > 0$. It is evident that the necessary and sufficient existence condition of solutions of equation (42) is negativity of the function $f(\nu)$ at the right-hand end point of the interval:

$$f(\nu_{\mathrm{L}}) < 0. \tag{45}$$

If the solution exists then it is unique.

Now we analyse whether condition (45) is satisfied for each of the cases enumerated in table 1.

Case (i), $\mu_1 < \mu_2$. Condition (45) is not satisfied:

$$f(\nu_{\mathrm{L}}) = \frac{a}{\sqrt{\mu_2}}(\mu_2-\mu_1) > 0.$$

Case (ii(a)), $\mu_1 > \mu_2$. Condition (45) is satisfied if $g \neq 0$:

$$f(\nu_{\mathrm{L}}) = -\frac{ag^2\sqrt{\mu_2}}{\mu_1^2-g^2} < 0.$$

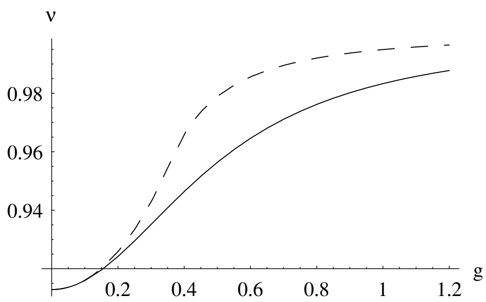

Figure 2. Dependence $\nu_S = \nu_S(g)$ (solid line) and $\nu_L = \nu_L(g)$ (dashed line) at $a = 1$, $\mu_1 = 1.2$, $\mu_2 = 1$.

Case (ii(b)), $\mu_1 > \mu_2$. We make use of the relation

$$\sqrt{\mu_2\left[a\mu_1 - (\mu_1^2 - g^2)\widetilde{\nu}_L^2\right](a - \mu_2\widetilde{\nu}_L^2)} = -\tfrac{1}{2}\left[\mu_1(a - \mu_2\widetilde{\nu}_L^2) + \mu_2(a - \mu_1\widetilde{\nu}_L^2)\right], \tag{46}$$

which is derived in appendix B. We have

$$\begin{aligned} f(\nu_L) &= \sqrt{\mu_2}(a - \mu_1\widetilde{\nu}_L^2) + \sqrt{\left[a\mu_1 - (\mu_1^2 - g^2)\widetilde{\nu}_L^2\right](a - \mu_2\widetilde{\nu}_L^2)} \\ &= \frac{1}{\sqrt{\mu_2}}\left\{\mu_2(a - \mu_1\widetilde{\nu}_L^2) - \frac{1}{2}\left[\mu_1(a - \mu_2\widetilde{\nu}_L^2) + \mu_2(a - \mu_1\widetilde{\nu}_L^2)\right]\right\} \\ &= \frac{a}{2\sqrt{\mu_2}}(\mu_2 - \mu_1) < 0. \end{aligned}$$

In this case condition (45) is satisfied too.

Thus, dispersion equation (42) has a solution and it is unique if the following simple relations for parameters of the contacting media are fulfilled:

$$\mu_1 > \mu_2, \qquad g \neq 0. \tag{47}$$

Surface electromagnetic waves can be excited only if the adjoining halves of the magnetic crystal are both *anisotropic* (non-coincidence of μ_1 and μ_2) and *gyrotropic*. In the absence of at least one of these features, excitation of surface waves is impossible. Moreover, the first relation (47) indicates that the crystal has to be *negative* (i.e. in the limit $g \to 0$ the refractive index of body ordinary waves $n_o = \sqrt{\mu_1/a}$ is greater than the refractive indices of extraordinary waves $n_e = \sqrt{\mu_1\mu_2/[a(\mu_1\cos^2\theta + \mu_2\sin^2\theta)]}$, see equations (10)).

In figure 2 dependence of the dimensionless phase velocity ν_S of the surface electromagnetic wave on the gyration parameter g is plotted at fixed values of the parameters a, μ_1 and μ_2 (see formula (44)). Also the appropriate values of the limiting frequencies ν_L of the body waves depending on g are plotted according to the formulae of table 1.

Substituting solutions (44) of the dispersion equation into relations (24), we get plots of real and imaginary parts of decay coefficients η_1 and η_2 of partial waves depending on g (figure 3). If gyrotropy of the media is not large (g does not exceed value g_0, see formula (50) below) these decay coefficients are pure imaginary and the field intensities of the partial waves vanish exponentially when the point of observation moves away from the interface. As $g \to 0$ one of the coefficients (for definiteness, η_1) tends to zero, the second remains finite. This implies that for the limiting case of the non-gyrotropic medium, one of the partial waves becomes delocalized (weakly inhomogeneous), and the second remains strongly localized. But the weight factor $C_2 = -C_1\eta_1/\eta_2$ of the strongly localized wave here is much less than the weight factor C_1 of the weakly localized wave (see figure 4). Thus, as $g \to 0$ the surface

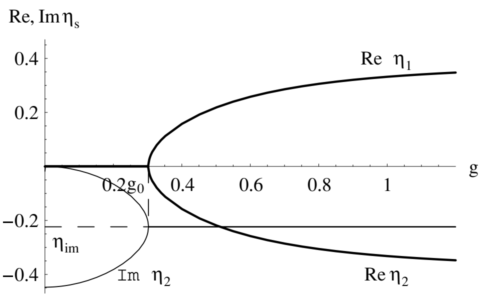

Figure 3. Dependences of Imη_s (light-faced lines) and Reη_s (bold-faced lines) on parameter g at $a = 1$, $\mu_1 = 1.2$, $\mu_2 = 1$.

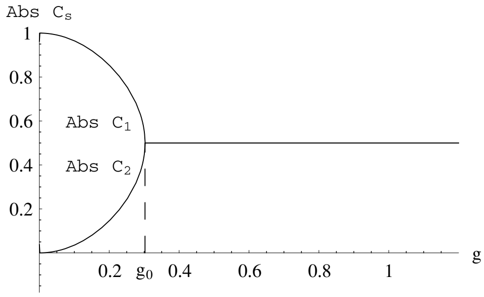

Figure 4. Relative weights $|C_1|$ and $|C_2|$ of partial waves ($a = 1$, $\mu_1 = 1.2$, $\mu_2 = 1$).

wave is transformed into a body wave and this is confirmed by the small difference between ν_S and ν_L at $g \approx 0$ (figure 2).

At $g > g_0$ the pattern of change in the partial wave intensities when moving away from the interface becomes quite different. In this case coefficients η_1 and η_2 have non-zero real parts which are opposite in sign. The amplitudes of the partial waves obey the 'decaying sine' law while $|z|$ is increased. Imaginary parts of η_1 and η_2 are the same and, furthermore, stop depending on g and remain equal to η_{im} (figure 3). Relative weights of the partial wave are the same and equal to $\frac{1}{2}$ (figure 4).

Let us find the critical value g_0 that corresponds to reconstruction of the wave solutions. At $g = g_0$ parameters η_1 and η_2 coincide and setting the radical expression in (24) to zero gives

$$4\mu_2^2 g^2 \nu^4 - 4a\mu_2 g^2 \nu^2 + a^2(\mu_1 - \mu_2)^2 = 0. \tag{48}$$

Solving (43) and (48) with respect to ν^2, we have

$$\nu^2 = \frac{a(\mu_1 + 3\mu_2)}{4\mu_1\mu_2}. \tag{49}$$

Moreover, substitution of relation (49) into (48) gives the desired value g_0:

$$g = g_0 = 2\mu_1\sqrt{\frac{\mu_1 - \mu_2}{3(\mu_1 + 3\mu_2)}}. \tag{50}$$

The imaginary parts of the decay coefficients η_1 and η_2 are obtained by substitution of (49) into (24): Im $\eta_{1,2} = \eta_{\mathrm{im}} = -\frac{1}{2}\sqrt{(\mu_1 - \mu_2)/\mu_2}$.

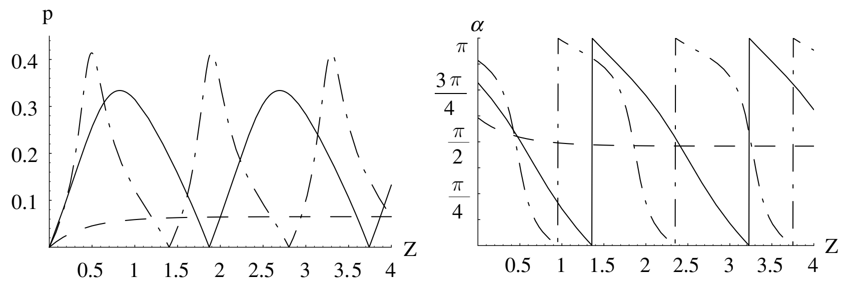

Figure 5. Dependence of parameters p and α on the normalized distance Z from the interface ($a = 1$, $\mu_1 = 1.2$, $\mu_2 = 1$, dashed lines—$g = 0.2$, solid lines—$g = 0.6$, dot-and-dash lines—$g = 1.3$). Critical value is $g_0 = 0.3024$.

Now we determine polarization of the surface wave. Using arbitrariness of choice of weight factor C_1, we put $C_1 = \eta_2/(\eta_1 - \eta_2)$. Then $C_2 = -\eta_1/(\eta_1 - \eta_2)$. The intensities of magnetic and electric field at the interface are determined by the equations

$$\boldsymbol{H}^0 = \sum_{s=1}^{2} C_s \boldsymbol{H}_s^0, \qquad \boldsymbol{E}^0 = \sum_{s=1}^{2} C_s \boldsymbol{E}_s^0.$$

According to formulae (25) and (26) subjected to (32), we have

$$\begin{aligned} \boldsymbol{H}^0 &= a^2\eta_1\eta_2(\eta_1 + \eta_2)\boldsymbol{q} + \mathrm{i}g\nu^2(a - \mu_2\nu^2)\boldsymbol{a}, \\ \boldsymbol{E}^0 &= a\nu[\mathrm{i}g(a - \mu_2\nu^2)\boldsymbol{q} - a\mu_2\eta_1\eta_2(\eta_1 + \eta_2)\boldsymbol{a}]. \end{aligned} \tag{51}$$

The coefficients of vectors $\boldsymbol{q}$ and $\boldsymbol{a}$ in (51) are pure imaginary. Consequently, at the interface $z = 0$ the wave is transverse ($\boldsymbol{b}\boldsymbol{H}^0 = \boldsymbol{b}\boldsymbol{E}^0 = 0$) and linearly polarized.

To calculate polarization outside of the interface one should use formulae (4). It turns out, that at $z \neq 0$ the surface wave is elliptically polarized, the semi-minor axis of the polarization ellipse being oriented along unit vector $\boldsymbol{b}$ of the direction of wave propagation and the semi-major axis lying in the plane passing through vectors $\boldsymbol{q}$ and $\boldsymbol{a}$. Let us introduce polarization parameter p which is the ratio of the length of the semi-minor axis to the length of the semi-major axis ($p = 0$ corresponds to linear polarization, $p = 1$ corresponds to circular polarization). Also we characterize inclination of the polarization ellipse to the interface by angle α ($0 \leqslant \alpha \leqslant \pi$): $\boldsymbol{d} = \boldsymbol{a}\cos\alpha + \boldsymbol{q}\sin\alpha$, where $\boldsymbol{d}$ is the unit vector directed along the semi-major axis. In figure 5 for different g, a plot of parameters p and α for the electric field $\boldsymbol{E}$ versus the normalized distance from the interface $Z = |z|/\lambda_0$ is made. Here $\lambda_0 = 2\pi c/\omega$ is the wave length in vacuum. The type of these plots is essentially different for values of g less than g_0 (50) and for values of g that exceed g_0. In the first case p and α change monotonically, tending to some limiting values at $|z| \to \infty$. In the second case they change periodically (it is clear that here the polarization ellipse continuously rotates about $\boldsymbol{b}$ while $|z|$ is increased). The parameters change identically for both contacting media and only directions of traversal of the polarization ellipses are opposite for each of the media.

The time averaged energy density of the electromagnetic field and the Poynting vector are calculated by the formulae

$$w = \frac{1}{16\pi}\left(\frac{1}{a}\boldsymbol{E}^*\boldsymbol{E} + \boldsymbol{H}^*\mu\boldsymbol{H}\right), \qquad \boldsymbol{S} = \frac{c}{8\pi}\mathrm{Re}(\boldsymbol{E} \times \boldsymbol{H}^*) \tag{52}$$

(for the field in half-space $z > 0$ all quantities in (52) are replaced by primed ones). Let w^0 and $\boldsymbol{S}^0$ is the energy density and Poynting vector near the interface, respectively. Dependence

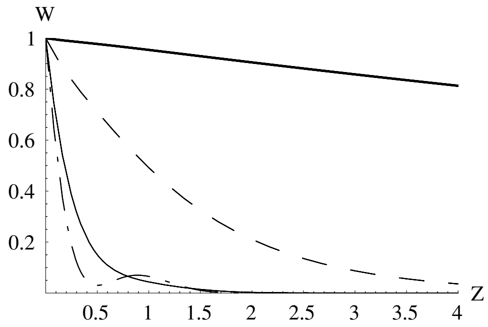

Figure 6. The normalized time averaged energy density of the surface wave as a function of distance Z ($a = 1$, $\mu_1 = 1.2$, $\mu_2 = 1$, solid bold-faced line—$g = 0.05$, dashed line—$g = 0.2$, solid light-faced line—$g = 0.6$, dot-and-dash line—$g = 1.3$).

of the normalized energy density $W = w/w^0$ on distance $Z = |z|/\lambda_0$ for different g is shown in figure 6. As a numerical calculation shows the energy-flux density $\boldsymbol{S}$ is parallel to vector $\boldsymbol{b}$ at any z. Furthermore, dependence of the normalized energy-flux density $|\boldsymbol{S}|/|\boldsymbol{S}^0|$ on Z completely coincides with that of the normalized energy density W.

Now consider a practically important case of weakly gyrotropic media ($g \approx 0$). We represent the quantity ν^2 as an expansion in a power series with respect to g^2: $\nu^2 = c_0 + c_2 g^2 + c_4 g^4 + \cdots$ and substitute this expansion into the dispersion equation (43). Keeping in (43) the powers of g not higher than the fourth, we get

$$\nu^2 = \nu_{\mathrm{S}}^2 \approx \frac{a}{\mu_1}\left[1 + \frac{g^2}{\mu_1^2} - \frac{2\mu_2 - \mu_1}{\mu_1^4(\mu_1 - \mu_2)} g^4\right]. \tag{53}$$

The squared reduced frequency ν_{S}^2 of the surface wave differs from the squared limiting frequency ν_{L}^2 of the appropriate body wave only in terms with the fourth and higher powers of the parameter g:

$$\nu_{\mathrm{L}}^2 = \frac{a\mu_1}{\mu_1^2 - g^2} \approx \frac{a}{\mu_1}\left(1 + \frac{g^2}{\mu_1^2} + \frac{g^4}{\mu_1^4}\right).$$

Similarly, substituting equation (53) into (21), (22) and taking into consideration the expansion $\eta^2 = d_0 + d_2 g^2 + d_4 g^4 + \cdots$, we estimate the decay coefficients of partial waves:

$$\begin{aligned}
\eta_1 &\approx -\mathrm{i}\frac{g^2}{\mu_1^2}\sqrt{\frac{\mu_2}{\mu_1 - \mu_2}}, \\
\eta_2 &\approx -\mathrm{i}\sqrt{\frac{\mu_1 - \mu_2}{\mu_2} - \frac{2g^2}{\mu_1^2} + \frac{3\mu_2 - 2\mu_1}{\mu_1^4(\mu_1 - \mu_2)} g^4} \approx -\mathrm{i}\left(\sqrt{\frac{\mu_1 - \mu_2}{\mu_2}} - \frac{g^2}{\mu_1^2}\sqrt{\frac{\mu_2}{\mu_1 - \mu_2}}\right).
\end{aligned} \tag{54}$$

The energy density distribution of the electromagnetic field at $g \approx 0$ is determined substantially by the contribution of the weakly inhomogeneous partial wave with decay coefficient η_1 and is proportional to $\exp(-2\mathrm{i}k\eta_1|z|) = \exp(-4\pi\mathrm{i}\eta_1 Z/\nu)$, $Z = |z|/\lambda_0$. Taking into account the first of formulae (54) and the approximate formula $\nu \approx \sqrt{a/\mu_1}$, we obtain an estimate of a characteristic penetration depth of the surface wave, for which the wave intensity is $\frac{1}{\mathrm{e}}$ of the intensity at the interface:

$$\widetilde{Z} = \frac{1}{4\pi g^2}\sqrt{\frac{a\mu_1^3(\mu_1 - \mu_2)}{\mu_2}}.$$

Thus, the penetration depth is inversely proportional to the squared gyration parameter g^2.

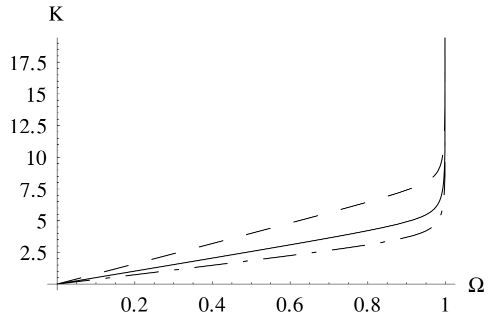

Figure 7. Dispersion dependences ($a = 1$, dashed line—$\beta = 0.2$, solid line—$\beta = 0.5$, dot-and-dash line—$\beta = 1$).

6. Interface of ferromagnetics with the axis of easy magnetization

The relations obtained above concern arbitrary uniaxial magnetic gyrotropic crystals whose material parameters a, μ_1, μ_2 and g at frequency ω of surface wave are regarded as given. Now consider surface polaritons at the interface of the uniformly magnetized ferromagnetics with axis of easy magnetization [28, 32] and magnetization vectors $\boldsymbol{M}$ and $\boldsymbol{M}'$ which are perpendicular to the interface and directed oppositely ($\boldsymbol{M}' = -\boldsymbol{M}$). In the absence of dissipation, the ferromagnetic in half-space $z < 0$ is described by a magnetic permeability tensor μ (2), and the ferromagnetic in half-space $z > 0$ by the complex conjugate tensor μ' (3). If an external stationary magnetic field is absent then the parameters involved in (2), (3) as a function of frequency ω are calculated by the formulae

$$\mu_1 = 1 + \frac{4\pi}{\beta}\frac{\omega_M^2}{\omega_M^2 - \omega^2}, \qquad \mu_2 = 1, \qquad g = -\frac{4\pi}{\beta}\frac{\omega\omega_M}{\omega_M^2 - \omega^2}, \tag{55}$$

where $\omega_M = \Gamma\beta M$ is a resonance frequency, Γ is the magnetomechanical ratio, $M = |\boldsymbol{M}|$ is a magnetization and β is an anisotropy factor. Here we do not take into consideration the presence of a transition layer between ferromagnetics[1] owing to rearrangement of domain structures near the interface, and suppose that the interface is sharp.

Since parameters μ_1 and μ_2 are regarded throughout to be positive, we do not consider frequencies of surface waves in the interval $(\omega_M, \omega_M\sqrt{1 + 4\pi/\beta})$, when μ_1 is negative. It is easy to see that according to the existence condition (47), a surface wave at the interface of ferromagnetics can be excited if its frequency ω does not exceed the value ω_M. In this context the decay coefficients of partial waves η_1 and η_2 are pure imaginary at any values of ω_M and β. That corresponds to exponential decay of these waves while distance $|z|$ from the interface is increased. Really, one can show that g^2 does not exceed a critical value g_0^2 (50). For this purpose we introduce a normalized frequency $\Omega = \omega/\omega_M$ ($0 < \Omega < 1$) and the designation $\delta = 4\pi/\beta$, then we find a lower bound of $g_0^2 - g^2$, substituting (55) into (50):

$$\begin{aligned} g_0^2 - g^2 &= \frac{\delta}{3(1-\Omega^2)^2[4(1-\Omega^2)+\delta]}\{4(1-\Omega^2+\delta)^2 - 3\delta\Omega^2[4(1-\Omega^2)+\delta]\} \\ &> \frac{\delta}{3(1-\Omega^2)^2[4(1-\Omega^2)+\delta]}\{4(1-\Omega^2+\delta)^2 - 3\delta[4(1-\Omega^2)+\delta]\} \\ &= \frac{\delta[2(1-\Omega^2)-\delta]]^2}{3(1-\Omega^2)^2[4(1-\Omega^2)+\delta]} \geqslant 0. \end{aligned}$$

[1] Study of surface polaritons taking into account a transition layer between gyrotropic crystals is a subject for future publication.

Plots of dispersion dependences for a series of values of anisotropy factor β are presented in figure 7. In this figure normalized frequencies Ω are the abscissae, and normalized wave numbers $K = ck/\omega_M = \Omega/\nu$ the ordinates. The reduced frequency ν is found by formula (44) when equations (55) are substituted into it. One can see that as $\omega \to \omega_M$ the wave number k becomes infinite ($\nu \to 0$). Thus, at frequencies close to resonance frequency ω_M, the surface wave is in essence magnetostatic (some other illustrations of excitation of body and surface magnetostatic waves in magnetically ordered media are given, e.g., in [32]).

7. Concluding remarks

Existence conditions (47) for surface magnetic polaritons are simple and can be satisfied in a broad frequency band. In particular, for transparent uniformly magnetized uniaxial ferromagnetics this band spreads from 0 to ω_M. Separate study is needed for the case of negative values of μ_1 and μ_2. Energy absorption can be considered if parameters μ_1 and μ_2 are taken complex.

By analogy with the term 'anisotropy-driven polaritons' introduced in [7] to designate singular surface electromagnetic waves in anisotropic dielectric media, we propose the term 'magnetic gyrotropy-driven polaritons' for the surface waves considered in this paper. Existence of such waves is possible given gyrotropic properties and magnetic anisotropy (non-coincidence of μ_1 and μ_2) of bordering media.

Expressions (38) and (39) for the surface impedance tensors obtained in this work can be applied for studying the surface waves at the interface of a magnetic gyrotropic crystal with magnetic permeability tensor (2) and isotropic medium with scalar magnetic permeability μ' too. For this purpose one should put $g = 0$, $\mu_1 = \mu_2 = \mu'$ in (39) and then substitute tensors γ and γ' into the dispersion equation (37).

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Appendix A. Surface impedance tensors γ and γ'

As was shown in paper [14], the surface impedance tensors for surface waves propagating along the interface of linear bianisotropic media have the form

$$\gamma = \frac{1}{\nu} Q^-(-\mathrm{i}I - S), \qquad \gamma' = \frac{1}{\nu} Q'^-(\mathrm{i}I - S'), \tag{A.1}$$

where $I = -\boldsymbol{q}^\times \boldsymbol{q}^\times = 1 - \boldsymbol{q} \otimes \boldsymbol{q}$ is the operator of orthogonal projection onto the interface plane with unit normal $\boldsymbol{q}$, sign $^-$ marks operation of pseudoinversion of a tensor (i.e. inversion in the two-dimensional subspace of three-dimensional space which is orthogonal to $\boldsymbol{q}$, e.g., $QQ^- = Q^-Q = I$). Tensor γ relates to the medium situated in half-space $z = \boldsymbol{qr} < 0$, and tensor γ' to the medium situated in half-space $z > 0$. Planar tensors Q and S involved in the first of formulae (A.1) have the following integral representation:

$$Q = -\frac{1}{\pi} \int_0^\pi (e_2\, e_2)^- \,\mathrm{d}\phi, \qquad S = -\frac{1}{\pi} \int_0^\pi (e_2\, e_2)^- (e_2\, e_1) \,\mathrm{d}\phi, \tag{A.2}$$

where tensorial bilinear forms $(\boldsymbol{uv})$ of vectorial arguments $\boldsymbol{u}$ and $\boldsymbol{v}$ for magnetic crystals with the dielectric permittivity and magnetic permeability tensors ε and μ are calculated in the following way[2]:

$$(\boldsymbol{uv}) = \boldsymbol{I}\boldsymbol{u}^{\times}\varepsilon^{-1}\boldsymbol{v}^{\times}\boldsymbol{I} + \boldsymbol{ub}\otimes\boldsymbol{bv}\nu^{2}\boldsymbol{I}\mu\boldsymbol{I} - (\boldsymbol{a}\varepsilon^{-1}\boldsymbol{a} - \nu^{2}\boldsymbol{q}\mu\boldsymbol{q})^{-1}$$
$$\times(\boldsymbol{I}\boldsymbol{u}^{\times}\varepsilon^{-1}\boldsymbol{a} + \nu^{2}\boldsymbol{I}\mu\boldsymbol{q}\otimes\boldsymbol{bu})\otimes(\boldsymbol{a}\varepsilon^{-1}\boldsymbol{v}^{\times}\boldsymbol{I} - \nu^{2}\boldsymbol{vb}\otimes\boldsymbol{q}\mu\boldsymbol{I}). \tag{A.3}$$

In (A.3) $\boldsymbol{b}$ is the unit vector directed along the propagation direction of the surface wave, and $\boldsymbol{a} = \boldsymbol{b}\times\boldsymbol{q}$. Unit vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are defined in the following way:

$$\boldsymbol{e}_1 = \boldsymbol{b}\cos\phi + \boldsymbol{q}\sin\phi, \qquad \boldsymbol{e}_2 = -\boldsymbol{b}\sin\phi + \boldsymbol{q}\cos\phi.$$

Tensors Q' and S' are calculated by the formulae analogous to (A.2)–(A.3) with replacement of tensors ε and μ in (A.3) by tensors ε' and μ' that relate to the medium $z > 0$.

Let us calculate tensorial bilinear forms $(\boldsymbol{e}_2\,\boldsymbol{e}_2)$ and $(\boldsymbol{e}_2\,\boldsymbol{e}_1)$ for the medium characterized by scalar inverse dielectric permittivity $a = \varepsilon^{-1}$ and tensorial magnetic permeability (2) ($\boldsymbol{c} = \boldsymbol{q}$). For that, in (A.3) we put vector $\boldsymbol{u}$ equal to $\boldsymbol{e}_2$, and vector $\boldsymbol{v}$ equal to $\boldsymbol{e}_2$ or $\boldsymbol{e}_1$, respectively. We obtain

$$(\boldsymbol{e}_2\,\boldsymbol{e}_2) = \nu^2\left(\frac{a\mu_2\cos^2\phi}{a-\mu_2\nu^2} + \mu_1\sin^2\phi\right)\boldsymbol{b}\otimes\boldsymbol{b}$$
$$+\mathrm{i}g\nu^2\sin^2\phi(\boldsymbol{b}\otimes\boldsymbol{a} - \boldsymbol{a}\otimes\boldsymbol{b}) - [a\cos^2\phi + (a-\mu_1\nu^2)\sin^2\phi]\boldsymbol{a}\otimes\boldsymbol{a}, \tag{A.4}$$

$$(\boldsymbol{e}_2\,\boldsymbol{e}_1) = \sin\phi\cos\phi\left[\left(\frac{a^2}{a-\mu_2\nu^2} - a - \mu_1\nu^2\right)\boldsymbol{b}\otimes\boldsymbol{b} - \mathrm{i}g\nu^2(\boldsymbol{b}\otimes\boldsymbol{a} - \boldsymbol{a}\otimes\boldsymbol{b}) - \mu_1\nu^2\boldsymbol{a}\otimes\boldsymbol{a}\right]. \tag{A.5}$$

The planar tensor $(\boldsymbol{e}_2\,\boldsymbol{e}_2)$ in the two-dimensional subspace of the interface is associated with the 2×2 matrix with elements that equal the coefficients of dyads $\boldsymbol{b}\otimes\boldsymbol{b}$, $\boldsymbol{b}\otimes\boldsymbol{a}$, $\boldsymbol{a}\otimes\boldsymbol{b}$, $\boldsymbol{a}\otimes\boldsymbol{a}$ in formula (A.4). Its determinant (or trace of the tensor adjoined to $(\boldsymbol{e}_2\,\boldsymbol{e}_2)$ [16]) equals

$$\overline{(\boldsymbol{e}_2\,\boldsymbol{e}_2)}_t = -\frac{\nu^2}{a-\mu_2\nu^2}(A\cos^4\phi + B\sin^2\phi\cos^2\phi + C\sin^4\phi),$$

where quantities A, B and C are given by formulae (22). Pseudoinverse tensor $(\boldsymbol{e}_2\,\boldsymbol{e}_2)^-$ is associated with the inverse 2×2 matrix. Therefore

$$(\boldsymbol{e}_2\,\boldsymbol{e}_2)^- = (A\cos^4\phi + B\sin^2\phi\cos^2\phi + C\sin^4\phi)^{-1}\left\{\frac{1}{\nu^2}(a-\mu_2\nu^2)[a\cos^2\phi\right.$$
$$+(a-\mu_1\nu^2)\sin^2\phi]\boldsymbol{b}\otimes\boldsymbol{b} + \mathrm{i}g(a-\mu_2\nu^2)\sin^2\phi(\boldsymbol{b}\otimes\boldsymbol{a} - \boldsymbol{a}\otimes\boldsymbol{b})$$
$$\left.-[a\mu_2\cos^2\phi + \mu_1(a-\mu_2\nu^2)\sin^2\phi]\boldsymbol{a}\otimes\boldsymbol{a}\right\}. \tag{A.6}$$

Further we calculate tensors Q and S (A.2). Product $(\boldsymbol{e}_2\,\boldsymbol{e}_2)^-(\boldsymbol{e}_2\,\boldsymbol{e}_1)$ of tensors (A.6) and (A.5) includes odd powers of $\cos\phi$ and $\sin\phi$ only, so the integral of this product over ϕ is equal to zero: $S = 0$. Simultaneously we have

$$Q = -\frac{1}{\nu^2}(a-\mu_2\nu^2)[aJ_{20} + (a-\mu_1\nu^2)J_{02}]\boldsymbol{b}\otimes\boldsymbol{b}$$
$$-\mathrm{i}g(a-\mu_2\nu^2)J_{02}(\boldsymbol{b}\otimes\boldsymbol{a} - \boldsymbol{a}\otimes\boldsymbol{b}) + [a\mu_2J_{20} + \mu_1(a-\mu_2\nu^2)J_{02}]\boldsymbol{a}\otimes\boldsymbol{a},$$

where

$$(J_{02};\,J_{20}) = \frac{1}{\pi}\int_0^{\pi}\frac{(\sin^2\phi;\,\cos^2\phi)\,\mathrm{d}\phi}{A\cos^4\phi + B\sin^2\phi\cos^2\phi + C\sin^4\phi} \tag{A.7}$$

[2] Formula (A.3) can be obtained from the general equation (18) of paper [14], if the tensors $\hat{\alpha}$ and $\hat{\beta}$ corresponding to magnetoelectric coupling in constitutive equations are taken as zero.

(subscripts refer to powers of $\cos\phi$ and $\sin\phi$, respectively, in the numerator of the integrand fraction). Calculating the integrals (A.7), we obtain

$$Q = \frac{1}{\sqrt{a\Delta(\nu)}}\left\{-\frac{1}{\nu^2}\left[\frac{a-\mu_2\nu^2}{\sqrt{\mu_2}} + (a-\mu_1\nu^2)r\right]\boldsymbol{b}\otimes\boldsymbol{b} - \mathrm{i}gr(\boldsymbol{b}\otimes\boldsymbol{a} - \boldsymbol{a}\otimes\boldsymbol{b}) + (\sqrt{\mu_2}+\mu_1 r)\boldsymbol{a}\otimes\boldsymbol{a}\right\},$$

where

$$r = \sqrt{\frac{a-\mu_2\nu^2}{a\mu_1 - (\mu_1^2 - g^2)\nu^2}},$$

and the function $\Delta(\nu)$ is given by formula (40).

Finally, we calculate the determinant of the matrix associated with the planar tensor Q: $\overline{Q}_t = -r/(a\nu^2\sqrt{\mu_2})$, and find the pseudoinverse tensor Q^-:

$$Q^- = \frac{\sqrt{a}}{\sqrt{\Delta(\nu)}}\left\{-\nu^2\left(\mu_2\sqrt{\frac{a\mu_1 - (\mu_1^2-g^2)\nu^2}{a-\mu_2\nu^2}} + \mu_1\sqrt{\mu_2}\right)\boldsymbol{b}\otimes\boldsymbol{b} - \mathrm{i}g\nu^2\sqrt{\mu_2}(\boldsymbol{b}\otimes\boldsymbol{a} - \boldsymbol{a}\otimes\boldsymbol{b}) + \left[\sqrt{\mu_2}(a-\mu_1\nu^2) + \sqrt{\left[a\mu_1 - (\mu_1^2-g^2)\nu^2\right](a-\mu_2\nu^2)}\right]\boldsymbol{a}\otimes\boldsymbol{a}\right\}.$$

Since $S = 0$, then according to the first of formulae (A.1) the surface impedance tensor γ differs from Q^- only in the common multiplier $-\mathrm{i}/\nu$ and has the form (38), where functions $f(\nu)$ and $h(\nu)$ (36) are applied.

The magnetic permeability tensor μ' of the border medium in half-space $z > 0$ differs from μ only in the sign of the parameter g. Therefore an expression for the surface impedance tensor γ' is obtained from (38) with a change of g to $-g$. Moreover, comparison of the two formulae (A.1) ($S = S' = 0$) shows that the sign of expression (38) must be reversed too. As a result we arrive at expression (39).

Appendix B. Derivation of formula (46)

It is not difficult to make sure that the substitution of (22) into the expression $(B^2 - 4AC)/(4a^2)$ at $\nu = \widetilde{\nu}_L$ gives the left-hand side of equation (16). Therefore

$$\frac{1}{a^2}(B^2 - 4AC)\bigg|_{\nu=\widetilde{\nu}_L} = \Delta(\widetilde{\nu}_L)\Delta_1(\widetilde{\nu}_L) = 0. \tag{B.1}$$

In (B.1) $\Delta(\nu) = (B + 2\sqrt{AC})/a$ is the function introduced by formula (40), and $\Delta_1(\nu) = (B - 2\sqrt{AC})/a$. Let us show that $\Delta_1(\widetilde{\nu}_L) \neq 0$. Really, taking into account equation (17) and the inequality $g^2 > \mu_1^2(\mu_1 - \mu_2)/(\mu_1 + \mu_2)$ (see table 1, case (ii(b))), we have

$$\begin{aligned}\Delta_1(\widetilde{\nu}_L) &= \frac{1}{a}(B - 2\sqrt{AC})\bigg|_{\nu=\widetilde{\nu}_L} < \frac{B}{a}\bigg|_{\nu=\widetilde{\nu}_L} = a(\mu_1+\mu_2) - 2\mu_1\mu_2\widetilde{\nu}_L^2 \\ &= a\left[\mu_2 - \mu_1\sqrt{1 - \frac{1}{g^2}(\mu_1-\mu_2)^2}\right] \\ &< a\left[\mu_2 - \mu_1\sqrt{1 - \frac{\mu_1+\mu_2}{\mu_1^2(\mu_1-\mu_2)}(\mu_1-\mu_2)^2}\right] = a\left(\mu_2 - \mu_1\frac{\mu_2}{\mu_1}\right) = 0,\end{aligned}$$

i.e. $\Delta_1(\widetilde{\nu}_L) < 0$. Thus, it follows from (B.1) that $\Delta(\widetilde{\nu}_L) = 0$. Taking into consideration definition (40) of function $\Delta(\nu)$, we obtain formula (46).

References

[1] Agranovich V M 1982 *Surface Polaritons* ed V M Agranovich and D C Mills (Amsterdam: North-Holland)
[2] Boardman A D (ed) 1982 *Electromagnetic Surface Modes* (New York: Wiley)
[3] Barnes B and Sambles R 2006 *Phys. World* **19** (1) 17–21
[4] Marchevsky F N, Strizhevsky V L and Strizhevsky S V 1984 *Fiz. Tverd. Tela* **26** 1501–3
[5] D'yakonov M I 1988 *Sov. Phys.—JETP* **67** 714–6 (*Zh. Eksp. Theor. Fiz.* **94** 119–23)
[6] Averkiev N S and D'yakonov M I 1990 *Opt. Spectrosc.* **68** 653–6 (*Opt. Spektrosk.* **68** 1118–21)
[7] Walker D B, Glytsis E N and Gaylord T K 1998 *J. Opt. Soc. Am.* A **15** 248–60
[8] Darinskii A N 2001 *Crystallog. Rep.* **46** 842–4 (*Kristallografiya* **46** 916–8)
[9] Alshits V I and Lyubimov V N 2002 *Phys. Solid State* **44** 386–90 (*Fiz. Tverd. Tela* **44** 371–4)
[10] Alshits V I and Lyubimov V N 2002 *Phys. Solid State* **44** 1988–92 (*Fiz. Tverd. Tela* **44** 1895–9)
[11] Furs A N, Galynsky V M and Barkovsky L M 2005 *J. Phys. A: Math. Gen.* **38** 8083–101
[12] Furs A N and Barkovsky L M 1997 *Microwave and Opt. Technol. Lett.* **14** 301–5
[13] Furs A N and Barkovsky L M 1999 *J. Opt. A: Pure Appl. Opt.* **1** 109–15
[14] Galynsky V M, Furs A N and Barkovsky L M 2004 *J. Phys. A: Math. Gen.* **37** 5083–96
[15] Chadwick P and Smith G D 1977 *Foundations of the Theory of Surface Waves in Anisotropic Elastic Media (Adv. Appl. Mech. 17)* ed C-S Yih (New York: Academic) p 303
[16] Fedorov F I 1976 *Theory of Gyrotropy* (Minsk: Nauka i Teknika)
[17] Post E J 1962 *Formal Structure of Electromagnetics. General Covariance and Electromagnetics* (Amsterdam: North-Holland)
[18] Lakhtakia A (ed) 1990 *Selected Papers on Natural Optical Activity (SPIE Milestone Series* vol MS 15) (Washington: SPIE Optical Engineering Press)
[19] Yakovlev Yu M and Hendelev S Sh 1975 *Ferrite Monocrystals in Radio Electronics* (Moscow: Soviet Radio)
[20] Curie P 1895 *Ann. de Chim. et de Phys. (VII)* **5** 289–407
[21] Bloch F 1934 *Molecular Theory of Magnetism* (Kiev, Kharkov: Gosud. Nauch. Tekhn. Izd.)
[22] White R M 1983 *Quantum Theory of Magnetism* (Berlin: Springer)
[23] Mattis D C 1965 *The Theory of Magnetism. An Introduction to the Study of Cooperative Phenomena* (New York: Harper & Row)
[24] Feynman R P, Leighton R B and Sands M 1964 *The Feynman Lectures on Physics* vol 2 (Reading, MA: Addison-Wesley)
[25] Tamm I E and Mandelshtam L I 1925 *Math. Ann.* **95** 154
[26] Ginzburg V L (ed) 1975 *I E Tamm. Collected Works* vol 1 (Moscow: Nauka) p 33–61
[27] Tomil'chik L M and Fedorov F I 1959 *Kristallografiya* **4** 498–504
[28] Landau L D and Lifshitz E M 1982 *Electrodynamics of Continuous Media* (Moscow: Nauka)
[29] Landau L D and Lifshitz E M 1978 *Statistical Physics: Part II* (Moscow: Nauka)
[30] Silin V P and Rukhadze A A 1961 *Electromagnetic Properties of Plasma and Plasmalike Media* (Moscow: Gos. Atom Izdat.)
[31] Gurevich A G 1973 *Magnetic Resonance in Ferrites and Antiferromagnetics* (Moscow: Nauka)
[32] Kaganov M I, Pustylnik N B and Shalaeva T I 1997 *Usp. Fiz. Nauk* **167** 191–237
[33] Hartstein A, Burstein E, Maradudin A A, Brewer R and Wallis R F 1973 *J. Phys. C: Solid State Phys.* **6** 1266–76
[34] Filippov V V 1988 *Radiotek. Elektron.* **33** 1225–32
[35] Barkovsky L M, Borzdov G N and Lavrinenko A V 1987 *J. Phys. A: Math. Gen.* **20** 1095–106
[36] Borzdov G N 1997 *J. Math. Phys.* **38** 6328–66